Long heterochromatic paths in heterochromatic triangle free graphs*

He Chen and Xueliang Li

Center for Combinatorics and LPMC-TJKLC,

Nankai University, Tianjin 300071, P.R. China

chenhe@mail.nankai.edu.cn, lxl@nankai.edu.cn

Abstract

In this paper, graphs under consideration are always edge-colored. We consider long heterochromatic paths in heterochromatic triangle free graphs. Two kinds of such graphs are considered, one is complete graphs with Gallai colorings, i.e., heterochromatic triangle free complete graphs; the other is heterochromatic triangle free graphs with k-good colorings, i.e., minimum color degree at least k. For the heterochromatic triangle free graphs K_n, we obtain that for every vertex $v \in V(K_n)$, K_n has a heterochromatic v-path of length at least $d^c(v)$; whereas for the heterochromatic triangle free graphs G we show that if, for any vertex $v \in V(G)$, $d^c(v) \geq k \geq 6$, then G a heterochromatic path of length at least $\frac{3k}{4}$.

Keywords: Gallai coloring, k-Good coloring, Long heterochromatic path, Heterochromatic triangle free

AMS Subject Classification 2000: 05C38, 05C15

1. Introduction

We use Bondy and Murty [3] for terminology and notations not defined here and consider simple graphs only.

Let $G = (V, E)$ be a graph. By an *edge coloring* of G we will mean a function $C : E \rightarrow \mathbb{N}$, the set of natural numbers. If G is assigned such a coloring, then we say that G is an *edge-colored graph*. Denote the edge-colored graph by (G, C), and call $C(e)$ the *color* of the edge $e \in E$. We say that $C(uv) = \emptyset$ if $uv \notin E(G)$ for $u, v \in V(G)$. For a subgraph H of G, we denote $C(H) = \{C(e) \mid e \in E(H)\}$ and $c(H) = |C(H)|$. For a vertex v of G, we say that color i is *presented at* vertex v if some edge incident with v has color i. The *color degree* $d^c(v)$ is the number of different colors that are presented at v, and the *color neighborhood* $CN(v)$ is the set of different colors that are presented at v. All graphs considered in this paper are edge-colored.

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For a positive integer k, a coloring of a graph is called *k-good* if the minimum color degree of the graph is at least k. A path, or a cycle, or any subgraph is called *heterochromatic (rainbow, or multicolored)* if any two edges of it have different colors. A graph is called *heterochromatic triangle free* if it does not contain any (induced) heterochromatic triangles. If u and v are two vertices on a path P, uPv will denote the segment of P from u to v, whereas $vP^{-1}u$ will denote the same segment but from v to u. A path is called a *v-path* if it starts from the vertex v.

There are a lot of existing literature dealing with the existence of paths and cycles with special properties in edge-colored graphs. The heterochromatic Hamiltonian cycle or path problem was studied by Hahn and Thomassen [13], Rödl and Winkler (see [11]), Frieze and Reed [11], and Albert, Frieze and Reed [1]. In [2], Axenovich, Jiang and Tuza gave the range of the maximum k such that there exists a k-good coloring of $E(K_n)$ that contains no properly colored copy of a path with fixed number of edges, no heterochromatic copy of a path with fixed number of edges, no properly colored copy of a cycle with fixed number of edges and no heterochromatic copy of a cycle with fixed number of edges, respectively. In [9], Erdös and Tuza studied the heterochromatic paths in infinite complete graph K_ω. In [10], Erdös and Tuza studied the values of k, such that every k-good coloring of K_n contains a heterochromatic copy of F where F is a given graph with m edges ($m < n/k$). In [14], Manoussakis, Spyratos and Tuza studied (s,t)-cycles in 2-edge-colored graphs, where an (s,t)-cycle is a cycle of length $s+t$ and s consecutive edges are in one color and the remaining t edges are in the other color. In [15], Manoussakis, Spyratos, Tuza and Voigt studied conditions on the minimum number k of colors, sufficient for the existence of given types (such as families of internally pairwise vertex-disjoint paths with common endpoints, Hamiltonian paths and Hamiltonian cycles, cycles with a given lower bound of their length, spanning trees, stars, and cliques) of properly edge-colored subgraphs in a k-edge-colored complete graph. In [8], Chou, Manoussakis, Megalaki, Spyratos and Tuza showed that for a 2-edge-colored graph G and three specified vertices x, y and z, to decide whether there exists a color-alternating path from x to y passing through z is NP-complete. Many results in these mentioned papers were proved by using probabilistic methods.

In [2], Axenovich, Jiang and Tuza considered the local variation of anti-Ramsey problem. Namely, they studied the maximum integer k, denoted by $g(n,H)$, such that there exists a k-good edge coloring of K_n that does not contain any heterochromatic copy of a given graph H. They showed that for a fixed integer $k \geq 2$, $k-1 \leq g(n, P_{k+1}) \leq 2k-3$, i.e., if K_n is edge-colored by a $(2k-2)$-good coloring, then there must exist a heterochromatic path P_{k+1}, there exists a $(k-1)$-good coloring of K_n such that no heterochromatic path P_{k+1} exists.

In [4], the authors considered the long heterochromatic paths in general graphs with a k-good coloring and showed that if G is an edge-colored graph with $d^c(v) \geq k$ (color degree condition) for every vertex v of G, then G has a heterochromatic v-path of length at least $\lceil \frac{k+1}{2} \rceil$. In [5, 6], we got some better bound of the length of longest heterochromatic paths in general graphs with a k-good coloring.

Theorem 1.1 *[5] Let G be an edge-colored graph and $3 \leq k \leq 7$ an integer. Suppose that*

$d^c(v) \geq k$ *for every vertex* v *of* G. *Then* G *has a heterochromatic path of length at least* $k-1$.

Theorem 1.2 *[6] Let* G *be an edge-colored graph. If* $d^c(v) \geq k \geq 7$ *for any vertex* $v \in V(G)$, *then* G *has a heterochromatic path of length at least* $\lceil \frac{2k}{3} \rceil + 1$.

In [7], we showed that if $|CN(u) \cup CN(v)| \geq s$ (color neighborhood union condition) for every pair of vertices u and v of G, then G has a heterochromatic path of length at least $\lceil \frac{s+1}{2} \rceil$, and gave examples to show that the lower bound is best possible in some sense.

Some special edge colorings have also been studied, such as *Gallai colorings*, which is defined to be the edge colorings of complete graphs in which no heterochromatic triangles exist. In [12], Gyárfás and Simonyi studied the existence of special monochromatic spanning trees in such colorings, they also determined the size of largest monochromatic stars guaranteed to occur.

In this paper, we consider long heterochromatic paths in complete graphs K_n with Gallai colorings, i.e., heterochromatic triangle free complete graphs, and in heterochromatic triangle free graphs with k-good colorings. We obtain that if K_n is heterochromatic triangle free, then for every vertex $v \in V(K_n)$, K_n has a heterochromatic v-path of length at least $d^c(v)$. For the heterochromatic triangle free general graphs G, we show that if $d^c(v) \geq k \geq 6$ for any vertex $v \in V(G)$, then G has a heterochromatic path of length at least $\frac{3k}{4}$.

2. Heterochromatic triangle free complete graphs

In this section, we consider a heterochromatic triangle free complete graph G, and try to find a long heterochromatic path from it.

Theorem 2.1 *Suppose* G *is a heterochromatic triangle free complete graph. Then for every vertex* u *in* G, G *has a heterochromatic* u*-path of length at least* $d^c(u)$.

Proof. Let u be any vertex of G and let $d^c(u) = k$. Suppose $v_1, v_2, \ldots, v_k$ are k different neighbors of u such that the k edges $uv_1, uv_2, \ldots, uv_k$ all have distinct colors (see Figure 2.1).

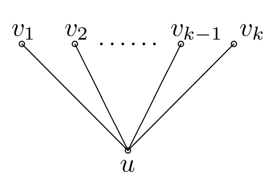

Figure 2.1

At first, we will construct a path P by the following inductive algorithm.

Algorithm

Step 1. If the two edges v_1v_2 and uv_1 have the same color, we let $w_1 = v_1$, $w_2 = v_2$; otherwise, we let $w_1 = v_2$, $w_2 = v_1$. Set $P_1 = w_1w_2$.

If **Step** $i-1$ is finished and we have obtained the path $P_{i-1} = w_1w_2\ldots w_i$. Then

Step i**.** If the two edges w_iv_{i+1} and uw_i have the same color, we let $w_{i+1} = v_{i+1}$.

Otherwise, if the color of the edge $w_{i-1}v_{i+1}$ is the same as the color of the edge uw_{i-1}, we let $w_{i+1} = w_i$ and $w_i = v_{i+1}$.

Otherwise, let j_0 be the maximum integer j such that the two edges w_jv_{i+1} and uv_{i+1} have the same color and the colors of the two edges $w_{j-1}v_{i+1}$ and uv_{i+1} are distinct. If all the i edges $w_1v_{i+1}, w_2v_{i+1}, \ldots, w_{i-1}v_{i+1}, uv_{i+1}$ have the same color, we set $j_0 = 1$.

If $j_0 = 1$, let $w_{i+1} = w_i$, $w_i = w_{i-1}$, $w_{i-1} = w_{i-2}$, …, $w_3 = w_2$, $w_2 = w_1$, $w_1 = v_{i+1}$. Otherwise, $2 \leq j_0 \leq i-1$, let $w_{i+1} = w_i$, $w_i = w_{i-1}$, …, $w_{j_0+1} = w_{j_0}$, $w_{j_0} = v_{i+1}$. Set $P_i = w_1w_2\ldots w_{i+1}$.

Continue the process till $i = k$ and we obtain the path $P = P_k$.

Then, we will prove the following claim about the path P obtained from the algorithm.

Claim. The path $P = w_1w_2\ldots w_k$ obtained from the algorithm is heterochromatic, the vertex set of path P is actually the set $\{v_1, v_2, \ldots, v_k\}$, and for each $1 \leq l \leq k-1$, the two edges w_lw_{l+1} and uw_l have the same color.

Proof. To prove the claim, we will show that for each i $(1 \leq i \leq k-1)$, the path $P_i = w_1w_2\ldots w_{i+1}$ we obtained after Step i satisfies that the vertex set of P_i is actually the set $\{v_1, v_2, \ldots, v_{i+1}\}$, and for each $1 \leq l \leq i$, the two edges w_lw_{l+1} and uw_l have the same color.

When $i = 1$, since there is no heterochromatic triangle in K_n, the edge v_1v_2 has the same color as the color of the edge uv_1 or uv_2. From Step 1, we can easily see that no matter which color the edge v_1v_2 has, the path $P = w_1w_2$ that is obtained after Step 1 contains actually two vertices v_1 and v_2, and the the two edges w_1w_2 and uw_1 have the same color.

Suppose Step $i-1$ has been finished, and the path $P = w_1w_2\ldots w_i$ we have obtained now contains actually i vertices $v_1, v_2, \ldots, v_i$, and for each $1 \leq l \leq i-1$, the two edges w_lw_{l+1} and uw_l have the same color, which is named to be color c_l. See Figure 2.2. Now we consider the path $P_i = w_1w_2\ldots w_{i+1}$ we obtained after Step i.

For convenience, if the path $P_i = w_1w_2\ldots w_{i+1}$ satisfies that the vertex set of P_i is actually the set $\{v_1, v_2, \ldots, v_{i+1}\}$, and for each $1 \leq l \leq i$, the two edges w_lw_{l+1} and uw_l have the same color, we say that P_i satisfies **Condition A**.

If the two edges w_iv_{i+1} and uw_i have the same color, then $w_{i+1} = v_{i+1}$. So, P_i obviously satisfies Condition A.

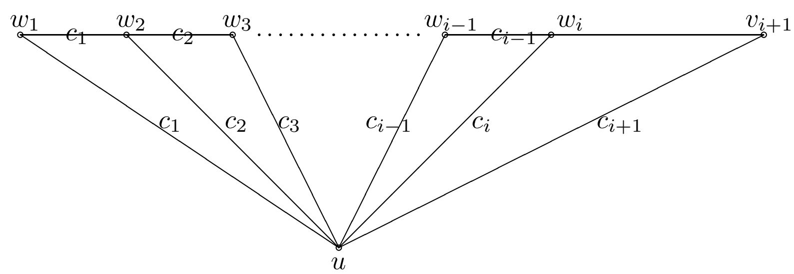

Figure 2.2

Otherwise, since the triangle uw_iv_{i+1} is not heterochromatic, the two edges w_iv_{i+1} and uv_{i+1} have the same color. In this case, if the color of the edge $w_{i-1}v_{i+1}$ is the same as the color of the edge uw_{i-1}, then $w_{i+1} = w_i$ and $w_i = v_{i+1}$ by Step i. Thus, P_i satisfies Condition A.

Now we consider the case when the two edges w_iv_{i+1} and uv_{i+1} have the same color, and the two edges $w_{i-1}v_{i+1}$ and uw_{i-1} have two distinct colors. Noticing that the triangle $uw_{i-1}v_{i+1}$ is not heterochromatic, we can conclude that the two edges $w_{i-1}v_{i+1}$ and uv_{i+1} have the same color.

If all the i edges $w_1v_{i+1}, w_2v_{i+1}, \ldots, w_{i-1}v_{i+1}, uv_{i+1}$ have the same color, we know that the two edges w_1v_{i+1} and uv_{i+1} have the same color and for each l $(1 \leq l \leq i-1)$, the two edges uw_l and w_lw_{l+1} have the same color. Thus, the path $v_{i+1}w_1w_2 \ldots w_i$ satisfies Condition A. This implies that when we set $w_{i+1} = w_i$, $w_i = w_{i-1}$, $w_{i-1} = w_{i-2}$, …, $w_3 = w_2$, $w_2 = w_1$, $w_1 = v_{i+1}$, the path $P_i = w_1w_2 \ldots w_iw_{i+1}$ satisfies Condition A.

Otherwise, we can find a maximum integer j, say j_0, such that the two edges w_jv_{i+1} and uv_{i+1} have the same color, and the colors of the two edges $w_{j-1}v_{i+1}$ and uv_{i+1} are distinct. It is clear from the discussion above that in this case, $2 \leq j_0 \leq i-1$. Then the vertex $w_{j_0-1} \in \{v_1, v_2, \ldots, v_i\}$. By the assumption that the colors of the k edges $uv_1, uv_2, \ldots, uv_k$ are all distinct, we know that the two edges uw_{j_0-1} and uv_{i+1} have two distinct colors. On the other hand, the two edges $w_{j_0-1}v_{i+1}$ and uv_{i+1} have two distinct colors. So we can conclude that the two edges $w_{j_0-1}v_{i+1}$ and uw_{j_0-1} have the same color, because the triangle $uw_{j_0-1}v_{i+1}$ is not heterochromatic. Then we can conclude that the path $w_1w_2 \ldots w_{j_0-1}v_{i+1}w_{j_0}w_{j_0+1} \ldots w_i$ satisfies Condition A. This implies that if we set $w_{i+1} = w_i$, $w_i = w_{i-1}$, …, $w_{j_0+1} = w_{j_0}$, $w_{j_0} = v_{i+1}$, the path $P_i = w_1w_2 \ldots w_iw_{i+1}$ satisfies Condition A. The Claim is thus proved by induction. ∎

Now we turn back to the proof of the theorem.

Since the path P we obtained from the algorithm satisfies all the conditions in the Claim, the color of the edge w_ku does not appear on the path P, and so the path w_1Pw_ku is a heterochromatic u-path of length k. The proof is thus complete. ∎

Using the theorem above, we can easily get the following result as a corollary.

Theorem 2.2 *Suppose G is a heterochromatic triangle free complete graph. If the maximum color degree among all the vertices in G is k, i.e., $\max_{v\in V(G)} d^c(v) = k$, then there is a heterochromatic path of length at least k in G.*

3. Heterochromatic triangle free general graphs

In this section, we consider long heterochromatic paths in a heterochromatic triangle free general graph. Before we give our main theorem, we would like to give some properties about this special kind of edge colored graphs.

Lemma 3.1 *Suppose G is a heterochromatic triangle free graph, and $P = u_0u_1 \dots u_l$ is a heterochromatic path of length $l \geq 5$. If the two edges u_0u_i and u_0u_j $(2 \leq i < i+1 < j \leq l)$ exist and their colors are distinct and do not appear on the path P, then either there exists an integer s, $i < s < j$, such that the edge u_0u_s does not exist, or there exist two integers s and t $(i < s < t \leq j)$, such that the two edges u_0u_s and u_0u_t have the same color.*

Proof. We will prove it by contradiction.

Suppose that we cannot get the conclusion, which implies that all the edges $u_0u_{i+1}, u_0u_{i+2}, \dots, u_0u_{j-1}, u_0u_j$ exist in G and they all have distinct colors.

First, we consider the triangle $u_0u_{j-1}u_j$. Since it is not heterochromatic, the color of the edge u_0u_j does not appear on the path P, and the two edges u_0u_{j-1} and u_0u_j have two distinct colors, we have that the two edges u_0u_{j-1} and $u_{j-1}u_j$ have the same color.

Now we consider the triangle $u_0u_{j-2}u_{j-1}$. As the two edges u_0u_{j-1} and $u_{j-1}u_j$ have the same color, and the path P is heterochromatic, we have that the two edges u_0u_{j-1} and $u_{j-2}u_{j-1}$ have two distinct colors. On the other hand, by the assumption, the triangle $u_0u_{j-2}u_{j-1}$ is not heterochromatic, and the two edges u_0u_{j-2} and u_0u_{j-1} have two distinct colors. So the two edges u_0u_{j-2} and $u_{j-2}u_{j-1}$ have the same color.

In the same way, we can get, orderly, the edge u_0u_{j-3} has the same color as the edge $u_{j-3}u_{j-2}$ has, …, the edge u_0u_{i+1} has the same color as the edge $u_{i+1}u_{i+2}$ has. Then the triangle $u_0u_iu_{i+1}$ is heterochromatic, a contradiction, which completes the proof. ∎

In a similar way, we can get the following property.

Lemma 3.2 *Suppose G is a heterochromatic triangle free graph, and $P = u_0u_1 \dots u_l$ is a heterochromatic path of length $l \geq 5$. If the edge u_0u_i exists and the color of it does not appear on the path P, then $i \geq 3$, and either there exists an integer s, $2 \leq s < i$, such that the edge u_0u_s does not exist, or there exist two integers s and t $(2 \leq s < t \leq i)$, such that the two edges u_0u_s and u_0u_t have the same color.*

Now we can state our main theorem.

Theorem 3.3 *Suppose G is a heterochromatic triangle free graph. If $d^c(v) \geq k \geq 6$ for any vertex $v \in V(G)$, then G has a heterochromatic path of length at least $\frac{3k}{4}$.*

Proof. Suppose $P = u_0u_1u_2 \ldots u_l$ is one of the longest heterochromatic paths in G. Assume that $CN(u_0)$ has s different colors not appearing on P, and $CN(u_l)$ has t different colors not appearing on P. Then there exist s different vertices $u_{x_1}, u_{x_2}, \ldots, u_{x_s}$ on the path P, where $2 \leq x_1 < x_2 < \ldots < x_s \leq l$, such that the colors of the s edges $u_0u_{x_1}, u_0u_{x_2}, \ldots, u_0u_{x_s}$ are all distinct and do not appear on P. There also exist t different vertices $u_{y_1}, u_{y_2}, \ldots, u_{y_t}$ on the path P, where $0 \leq y_1 < y_2 < \ldots y_t \leq l-2$, such that the colors of the t edges $u_{y_1}u_l, u_{y_2}u_l, \ldots, u_{y_t}u_l$ are all distinct and do not appear on P. Since there exists no heterochromatic triangle in G, we have $x_1 \geq 3$, $x_{i+1} > x_i + 1$ for $i = 1, 2, \ldots, s-1$; $y_t \leq l-3$, $y_{j+1} > y_j + 1$ for $j = 1, 2, \ldots, t-1$.

Since $k \geq 6$, we can conclude from Theorems 1.1 and 1.2 that the path P is of length $l \geq 5$. By Lemma 3.2, we have that

$$|\{C(u_0u_2), C(u_0u_3), \ldots, C(u_0u_{x_1})\}| \leq x_1 - 2.$$

We can also get from Lemma 3.1 that for any $1 \leq i \leq s-1$,

$$|\{C(u_0u_{x_i+1}), C(u_0u_{x_i+2}), \ldots, C(u_0u_{x_{i+1}-1}), C(u_0u_{x_{i+1}})\}| \leq x_{i+1} - x_i - 1.$$

So

$$\begin{aligned} & |\{C(u_0u_1), C(u_0u_2), \ldots, C(u_0u_{l-1}), C(u_0u_l)\}| \\ \leq\; & |\{C(u_0u_1))\}| + |\{C(u_0u_2), C(u_0u_3), \ldots, C(u_0u_{x_1})\}| \\ & +|\{C(u_0u_{x_1+1}), C(u_0u_{x_1+2}), \ldots, C(u_0u_{x_2-1}), C(u_0u_{x_2})\}| \\ & +|\{C(u_0u_{x_2+1}), C(u_0u_{x_2+2}), \ldots, C(u_0u_{x_3-1}), C(u_0u_{x_3})\}| \\ & +\ldots \\ & +|\{C(u_0u_{x_{s-1}+1}), C(u_0u_{x_{s-1}+2}), \ldots, C(u_0u_{x_s-1}), C(u_0u_{x_s})\}| \\ & +|\{C(u_0u_{x_s+1}), \ldots, C(u_0u_{l-1}), C(u_0u_l)\}| \\ \leq\; & 1 + (x_1 - 2) + (x_2 - x_1 - 1) + (x_3 - x_2 - 1) + \ldots + (x_s - x_{s-1} - 1) + (l - x_s) \\ =\; & l - s. \end{aligned} \tag{3.1}$$

On the other hand, for any vertex v which is adjacent to u_0 but does not belong to the path P, the color of the edge u_0v is not same as the color of the edge $u_{y_j}u_{y_j+1}$ for any $1 \leq j \leq t$, for otherwise, $vu_0Pu_{y_j}u_lP^{-1}u_{y_j+1}$ is a heterochromatic path of length $l+1$, a contradiction. So we have $CN(u_0) \setminus \{C(u_0u_i) : 1 \leq i \leq l\} \subseteq C(P) \setminus \{C(u_{y_j}u_{y_j+1}) : 1 \leq j \leq t\}$, and then

$$|CN(u_0) \setminus \{C(u_0u_i) : 1 \leq i \leq l\}| \leq l - t. \tag{3.2}$$

From Inequalities 3.1 and 3.2, we have

$$\begin{aligned} k &\leq |CN(u_0)| \\ &\leq |CN(u_0) \setminus \{C(u_0u_i) : 1 \leq i \leq l\}| + |\{C(u_0u_i) : 1 \leq i \leq l\}| \\ &\leq (l - t) + (l - s) = 2l - s - t. \end{aligned} \tag{3.3}$$

On the other hand, since the color degrees of the two vertices u_0 and u_l are both at least k, and because of the assumption that P is one of the longest heterochromatic paths, we have that $l+s \geq k$, $l+t \geq k$. This implies that $s \geq k-l$ and $t \geq k-l$. Now we can get from Inequality 3.3 that

$$k \leq 2l - s - t \leq 2l - 2(k-l),$$

So, $4l \geq 3k$, and $l \geq \frac{3k}{4}$, and the proof is thus complete. ∎

4. Concluding remarks

Finally, we give examples to show that our lower bounds given in Theorems 2.1 and 2.2 are best possible.

Remark 4.1 *For any integer $k \geq 1$, there is a heterochromatic triangle free complete graph G_k with the color degree of every vertex v in G_k is k, i.e., $d^c(v) = k$, such that any longest heterochromatic v-path in G_k is of length k.*

Let G_k be an edge colored complete graph whose vertices are the ordered k-tuples of 0's and 1's. An edge is in color j $(1 \leq j \leq k)$ if and only if the first $j-1$ coordinates of its two ends are exactly the same and the j-th coordinates of its two ends are different.

It is not hard to see that there exist no heterochromatic triangles in G_k. Otherwise, suppose uvw is a heterochromatic triangle, where $u = (u_1, u_2, \ldots, u_k)$, $v = (v_1, v_2, \ldots, v_k)$ and $w = (w_1, w_2, \ldots, w_k)$, the edge uv is in color x, the edge vw is in color y, and the edge uw is in color z. Without loss of generality, we can assume that $1 \leq x < y < z \leq k$. Since the edge uv is in color x, we can conclude that the first $x-1$ coordinates of the two vertices u and v are exactly the same, and the x-th coordinates of u and v are different. Similarly, we have that the first $z-1$ coordinates of the two vertices u and w are exactly the same, and the z-th coordinates of u and w are different. Then the first $x-1$ coordinates of the two vertices v and w are exactly the same, and the x-th coordinates of v and w are different. So the edge vw is in color x, a contradiction.

It is obvious that for every vertex v in G_k, its color degree is k. So we can easily conclude that the longest heterochromatic v-path in G_k is of length at least k by Theorem 2.1. On the other hand, any longest heterochromatic path in G_k is not longer than k, since there are only k different colors used in this graph. Hence, the conclusion in the remark is true.

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